SECOND LAW OF THERMODYNAMICS
(Numerical Problem Solution)

e-content for B.Sc Physics (Honours)
B.Sc Part-I
Paper-II

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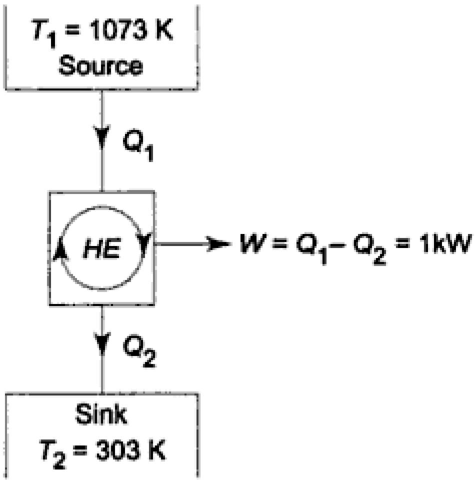
1.A cyclic heat engine operates between a source temperature of 800^0C and a sink temperature of 30^0C. What is the least rate of heat rejection per kW net output of the engine?

Solution: For a reversible engine, the rate of heat rejection will be minimum



$$\eta_{max} = \eta_{rev} = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{30 + 273}{800 + 273}$$
$$= 1 - 0.282 = 0.718$$

Now $$\frac{W_{net}}{Q_1} = \eta_{max} = 0.718$$

∴ $$Q_1 = \frac{1}{0.718} = 1.392 \text{ kW}$$

Now $$Q_2 = Q_1 - W_{net} = 1.392 - 1$$
$$= 0.392 \text{ kW}$$

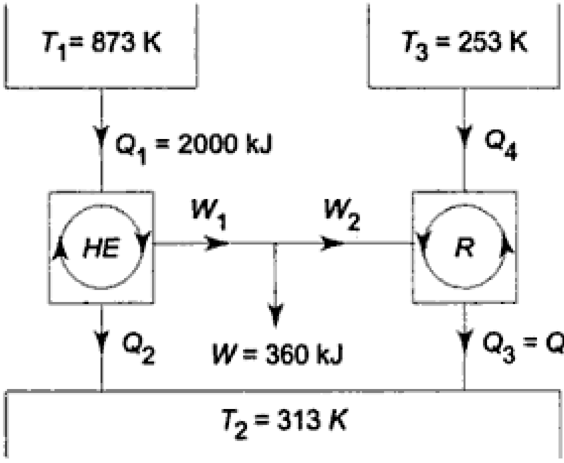
This is the least rate of heat rejection.

2. A reversible heat engine operates between two reservoirs at temperatures of 600^0C and 40^0C. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 40^0C and -20^0C. The heat transfer to the heat engine is 2000kJ and the net work output of the combined engine transfer plant is 360kJ.

(a) Evaluate the heat transfer to the refrigerant and the net heat transfer to the reservoir at 40^0C.

(b)Reconsider (a) given that the efficiency of the heat engine and the COP of the refrigerator are each 40% of their maximum possible values.

Solution: (a) Maximum efficiency of the heat engine cycle is given by



$$\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{313}{873} = 1 - 0.358 = 0.642$$

Again $\frac{W_1}{Q_1} = 0.642$

$\therefore$ $W_1 = 0.642 \times 2000 = 1284$ kJ

Maximum COP of the refrigerator cycle

$$(COP)_{max} = \frac{T_3}{T_2 - T_3} = \frac{253}{313 - 253} = 4.22$$

Also $COP = \frac{Q_4}{W_2} = 4.22$

Since $W_1 - W_2 = W = 360$ kJ

$\therefore$ $W_2 = W_1 - W = 1284 - 360 = 924$ kJ

$\therefore$ $Q_4 = 4.22 \times 924 = 3899$ kJ

$\therefore$ $Q_3 = Q_4 + W_2 = 924 + 3899 = 4823$ kJ

$Q_2 = Q_1 - W_1 = 2000 - 1284 = 716$ kJ

Heat rejection to the 40°C reservoir

$= Q_2 + Q_3 = 716 + 4823 = 5539$ kJ *Ans.* (a)

(b) Efficiency of the actual heat engine cycle

$\eta = 0.4\ \eta_{max} = 0.4 \times 0.642$

$\therefore$ $W_1 = 0.4 \times 0.642 \times 2000$

$= 513.6$ kJ

$\therefore$ $W_2 = 513.6 - 360 = 153.6$ kJ

COP of the actual refrigerator cycle

$$COP = \frac{Q_4}{W_2} = 0.4 \times 4.22 = 1.69$$

Therefore

$$Q_4 = 153.6 \times 1.69 = 259.6 \text{ kJ} \qquad \textit{Ans.} \text{ (b)}$$

$$Q_3 = 259.6 + 153.6 = 413.2 \text{ kJ}$$

$$Q_2 = Q_1 - W_1 = 2000 - 513.6 = 1486.4 \text{ kJ}$$

Heat rejected to the 40°C reservoir

$$= Q_2 + Q_3 = 413.2 + 1486.4 = 1899.6 \text{ kJ} \qquad \textit{Ans.} \text{ (b)}$$

3. It is proposed that solar energy be used to warm a large collector plate. This energy would, in turn, be transferred as heat to a fluid within a heat engine, and engine would reject energy as heat to the atmosphere. Experiments indicate that about 1880 kJ/m^{2}h of energy can be collected when the plate is operating at 90^{0}C. Estimate the minimum collector area that would be required for a plant producing 1kW of useful shaft power. The atmospheric temperature may be assumed to be 20^{0}C.

Solution: The maximum efficiency for the heat engine operating between the collector plate temperature and the atmospheric temperature is

$$\eta_{max} = 1 - \frac{T_2}{T_1} = 1 - \frac{293}{363} = 0.192$$

The efficiency of any actual heat engine operating between these temperatures would be less than this efficiency.

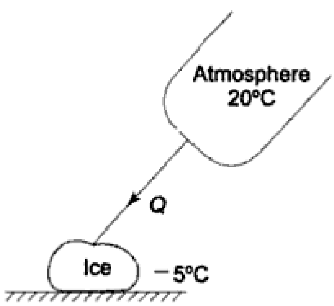
$$\therefore \qquad Q_{min} = \frac{W}{\eta_{max}} = \frac{1 \text{ kJ/s}}{0.192} = 5.21 \text{ kJ/s}$$

$$= 18{,}800 \text{ kJ/h}$$

$\therefore$ Minimum area required for the collector plate

$$= \frac{18{,}800}{1880} = 10 \text{ m}^2 \qquad \textit{Ans.}$$

4. One kg of ice at -50^{0}C is exposed to the atmosphere which is at 20^{0}C. The ice melts and comes into thermal equilibrium with the atmosphere. (a) Determine the entropy increases of the universe . (b) what is the minimum amount of work necessary to convert the water back into ice ar -5^{0}C? C_p of ice is 2.093 kJ/kgK and the latent heat of fusion of ice is 333.3 kJ/kg.



$= \text{Heat absorbed in solid phase} + \text{Latent heat}$
$\quad + \text{Heat absorbed in liquid phase}$
$= 1 \times 2.093 \times [0 - (-5)] + 1 \times 333.3 + 1 \times 4.187 \times (20 - 0)$
$= 427.5 \text{ kJ}$

Entropy change of the atmospher.

$$(\Delta S)_{atm} = -\frac{Q}{T} = -\frac{427.5}{293} = -1.46 \text{ kJ/K}$$

Entropy change of the system (ice) as it gets heated from –5°C to 0°C

$$(\Delta S_I)_{system} = \int_{268}^{273} mc_p \frac{dT}{T} = 1 \times 2.093 \ln \frac{273}{268} = 2.093 \times 0.0186$$
$$= 0.0389 \text{ kJ/K}$$

Entropy change of the system as ice melts at 0°C to become water at 0°C

$$(\Delta S_{II})_{system} = \frac{333.3}{273} = 1.22 \text{ kJ/K}$$

Entropy change of water as it gets heated from 0°C to 20°C

$$(\Delta S_{III})_{system} = \int_{273}^{293} mc_p \frac{dT}{T} = 1 \times 4.187 \ln \frac{293}{273} = 0.296 \text{ kJ/K}$$

Total entropy change of ice as it melts into water

$$(\Delta S)_{total} = \Delta S_I + \Delta S_{II} + \Delta S_{III}$$
$$= 0.0389 + 1.22 + 0.296$$
$$= 1.5549 \text{ kJ/K}$$

The entropy-temperature diagram for the system at -5^{0}C converts to water at 20^{0}C is shown in fig.

∴ Entropy increase of the universe

$$(\Delta S)_{univ} = (\Delta S)_{system} + (\Delta S)_{atm}$$
$$= 1.5549 - 1.46 = 0.0949 \text{ kJ/K} \qquad \textit{Ans.} \text{ (a)}$$

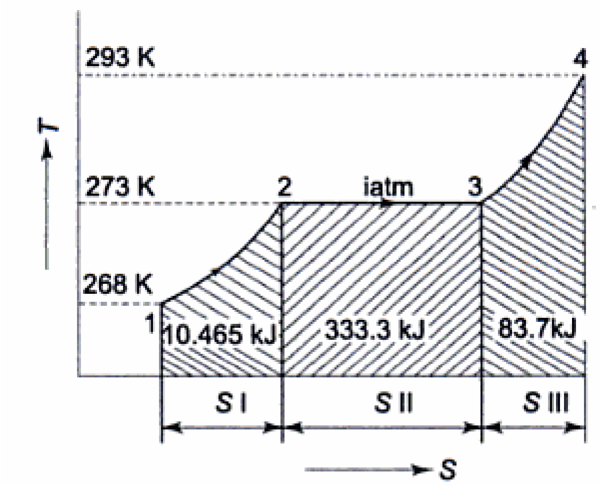


Fig.7.3.2

(b) To convert 1 kg of water at 20°C to ice at –5°C, 427.5 kJ of heat have to be removed from it, and the system has to be brought from state 4 to state 1 (Fig. Ex. 7.3.2). A refrigerator cycle, as shown in Fig. Ex. 7.3.3, is assumed to accomplish this.

The entropy change of the system would be the same, i.e. $S_4 - S_1$, with the only difference that its sign will be negative, because heat is removed from the system (Fig. Ex. 7.3.2).

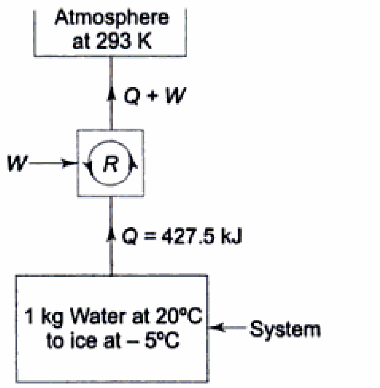


Fig.7.3.3

$$(\Delta S)_{\text{system}} = \underset{\text{(negative)}}{S_1 - S_4}$$

The entropy change of the working fluid in the refrigerator would be zero, since it is operating in a cycle, i.e.,

$$(\Delta S)_{\text{ref}} = 0$$

The entropy change of the atmosphere (positive)

$$(\Delta S)_{\text{atm}} = \frac{Q + W}{T}$$

∴ Entropy change of the universe

$$(\Delta S)_{\text{univ}} = (\Delta S)_{\text{system}} + (\Delta S)_{\text{ref}} + (\Delta S)_{\text{atm}}$$

$$= (S_1 - S_4) + \frac{Q + W}{T}$$

By the principle of increase of entropy

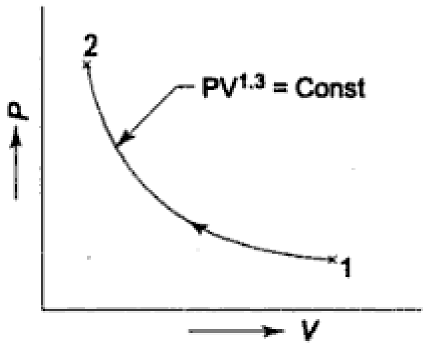
$$(\Delta S)_{\text{univ or isolated system}} \geq 0$$

$$\therefore \quad \left[(S_1 - S_4) + \frac{Q + W}{T}\right] \geq 0$$

5. A fluid undergoes a reversible adiabatic compression from 0.5 MPa, 0.2 m^3 to 0.05m^3 according to the law, $pv^{1.3}$=constant. Determine the change in enthalpy, internal energy and entropy, and the heat transfer and work transfer during the process.

Solution:

$$TdS = dH - VdP$$



$$dH = Vdp$$

$$p_1 = 0.5 \text{ MPa},\ V_1 = 0.2 \text{ m}^3$$

$$V_2 = 0.05 \text{ m}^3,\ p_1 V^n_1 = p_2 V_2^n$$

$$p_2 = p_1 \left(\frac{V_1}{V_2}\right)^n$$

$$= 0.5 \times \left(\frac{0.20}{0.05}\right)^{1.3} \text{ MPa}$$

$$= 0.5 \times 6.061 \text{ MPa}$$

$$= 3.0305 \text{ MPa}$$

$$p_1 V_1^n = pV^n$$

$$\therefore \qquad V = \left(\frac{p_1 V_1^n}{p}\right)^{1/n}$$

$$\int_{H_1}^{H_2} dH = \int_{p_1}^{p_2} V dp$$

$$H_2 - H_1 = \int_{p_1}^{p_2} \left[\left(\frac{p_1 V_1^n}{p}\right)^{1/n}\right] dp$$

$$= (p_1 V_1^n)^{1/n} \left(\frac{p_1^{1-1/n} - p_1^{1-n/n}}{1 - 1/n}\right)$$

$$= \frac{n(p_2 V_2 - p_1 V_1)}{n - 1}$$

$$= \frac{1.3(3030.5 \times 0.05 - 500 \times 0.2)}{1.3 - 1}$$

$$= 223.3 \text{ kJ}$$

$$H_2 - H_1 = (U_2 + p_2\, V_2) - (U_1 + p_1\, V_1)$$

$$= (U_2 - U_1) + (p_2\, V_2 - p_1 V_1)$$

$$U_2 - U_1 = (H_2 - H_1) - (p_2 V_2 - p_1 V_1)$$

$$= 223.3 - 51.53$$

$= 171.77$ kJ *Ans.*

$S_2 - S_1 = 0$ *Ans.*

$Q_{1\text{-}2} = 0$ *Ans.*

$$Q_{1\text{-}2} = U_2 - U_1 + W_{1\text{-}2}$$

$W_{1\text{-}2} = U_1 - U_2 = -171.77$ kJ *Ans.*

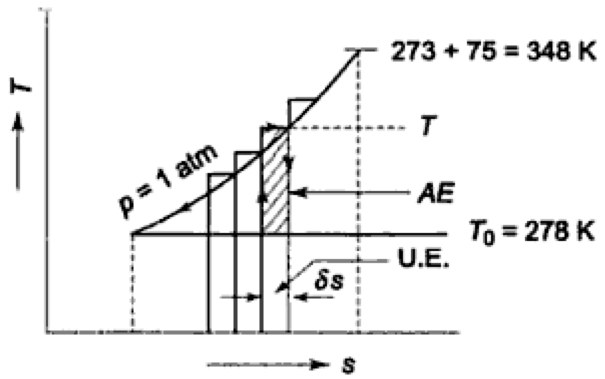
6. Calculate the available energy in 40kg of water at 75^0C with respect to the surrounding at 50^0C, the pressure of water being 1atm.

Solution:

If the water is cooled at a constant pressure of 1 atm from 75^0C to 5^0C (as shown in fig) the heat given up may be used as a source for a series of carnot engines each using the surrounding as a sink. It is assumed that the amount of energy received by any engine is small relative to that in the source and the temperature of the source doesnot change while heat is being exchanged with the engine.

Let us consider that the source has fallen to temperature T, at which level there operates a carnot engine which takes in heat at this temperature and rejects heat at T_0=278K. If del S is change in entropy water, the work obtainable is

$$\delta W = -m(T - T_0)\delta s$$



where δs is negative.

$$\delta W = -40(T - T_0)\frac{c_p \delta T}{T}$$

$$\therefore \qquad = -40c_p\left(1 - \frac{T_0}{T}\right)\delta T$$

With a very great number of engines in the series, the total work (maximum) obtainable when the water is cooled from 348K to 278K would be

$$W_{(\max)} = \text{A.E.} = -\lim \sum_{348}^{278} 40c_p\left(1 - \frac{T_0}{T}\right)\delta T$$

$$= \int_{278}^{348} 40c_p\left(1 - \frac{T_0}{T}\right)dT$$

$$= 40c_p\left[(348 - 278) - 278 \ln \frac{348}{278}\right]$$

$$= 40 \times 4.2\ (70 - 62)$$
$$= 1340 \text{ kJ} \qquad \textit{Ans.}$$
$$Q_1 = 40 \times 4.2\ (348 - 278)$$
$$= 11{,}760 \text{ kJ}$$
$$\text{U.E.} = Q_1 - W_{(max)}$$
$$= 11{,}760 - 1340 = 10{,}420 \text{ kJ}$$

7. Air enters a compressor at 1 bar, 30^0C, which is also the state of environment. It leaves at 3.5 bar, 141^0 and 90m/s. Neglecting inlet velocity and P.E. effect, determine (a) whether the compression is adiabatic or polytropic, (b) If not adiabatic, the polytropic index, (c) the isothermal efficiency, (d) the minimum work input and irreversibiity and (e) second law efficiency. Take C_p of air =1.0035kJ/kgK

Solution:
(a) After isentropic compression

$$\frac{T_{2s}}{T_1} = \left[\frac{p_2}{p_1}\right]^{(\gamma-1)/\gamma}$$

$$T_{2s} = 303\ (3.5)^{0.286} = 433.6 \text{ K} = 160.6°\text{C}$$

Since this temperature is higher than the given temperature of 141^0C, there is heat loss to the surroundings. The compression cannot be adiabatic. It must be polytropic.

(b)
$$\frac{T_2}{T_1} = \left[\frac{p_2}{p_1}\right]^{(n-1)/n}$$

$$\frac{141 + 273}{30 + 273} = 1.366 = \left(\frac{3.5}{1}\right)^{(n-1)/n}$$

$$\log 1.366 = \frac{n-1}{n} \log 3.5$$

$$1 - \frac{1}{n} = \frac{0.135}{0.544} = 0.248$$

$$n = 1.32978 = 1.33$$

(c) Actual work of compression

$$W_a = h_1 - h_2 - \frac{V_2^2}{2} = 1.0035\ (30 - 141) - \frac{90^2}{2} \times 10^{-3}$$
$$= -115.7 \text{ kJ/kg}$$

Isothermal work

$$W_T = \int_1^2 v\,dp - \frac{V_2^2}{2} = -RT_1 \ln \frac{p_2}{p_1} - \frac{V_2^2}{2}$$

$$= -0.287 \times 303 \ln (3.5) - \frac{90^2}{2} \times 10^{-3}$$

$$= -113 \text{ kJ/kg}$$

Isothermal efficiency:

$$\eta_T = \frac{W_T}{W_a} = \frac{113}{115.7} = 0.977 \text{ or } 97.7\% \qquad \textit{Ans.}$$

(d) Decrease in availability or exergy:

$$\psi_1 - \psi_2 = h_1 - h_2 - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2}$$

$$= c_p(T_1 - T_2) - T_0\left[R \ln \frac{p_2}{p_1} - c_p \ln \frac{T_2}{T_1}\right] - \frac{V_2^2}{2}$$

$$= 1.0035\,(30 - 141)$$

$$- 303\left[0.287 \ln 3.5 - 1.0035 \ln \frac{414}{303}\right] - \frac{90^2}{2000}$$

$$= -101.8 \text{ kJ/kg}$$

Minimum work input = – 101.8 kJ/kg *Ans.*

Irreversibility, $I = W_{rev} - W_a$

$$= -101.8 - (-115.7)$$

$$= 13.9 \text{ kJ/kg} \qquad \textit{Ans.}$$

(e) Second law efficiency,

$$\eta_{II} = \frac{\text{Minimum work input}}{\text{Actual work input}} = \frac{101.8}{115.7}$$

$$= 0.88 \text{ or } 88\% \qquad \textit{Ans.}$$