e-content for students

B. Sc.(honours) Part 2 paper 4

Subject:Mathematics

Topic:Principle of virtual work in two dimension

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Virtual Work

Definition

If there be no motion, no actual displacement is made, However we may allow the body to receive an imaginary displacement called the virtual displacement. Then the work done by the force in such a virtual displacement is called virtual work.

Principle of Virtual Work

Q:state and prove the principal of v irtual work for any system of force s in one plane

Statement. If a system of coplanar force acting on a rigid body be in equilibrium and if the body is imagined to undergo a slight displacement consistent with the geometrical conditions of the system, then the algebraic sum of virtual works done by the forces is zero.

Let $\vec{P_i}$ $(i=1, 2, 3, \ldots)$ be a system of coplanar forces acting at different points $\vec{r_i}$ $(i=1, 2, 3, \ldots)$ of a rigid body, the origin being an arbitrary point O. Since the forces are equivalent to a single force acting at an arbitrarily chosen point together with a couple, let the single force at O be $\vec{R}$ and the couple be $\vec{G}$.

Hence $$\vec{R} = \sum_{i=1} \vec{P_i} \quad \ldots (1)$$

and $$\vec{G} = \sum_{i=1} (\vec{r_i} \times \vec{P_i}). \quad \ldots (2)$$

Let us imagine the body to undergo a slight displacement (consistent with the geometrical conditions of the system), which consists of a small rotation $\vec{e}$ about O and a uniform translation $\vec{u}$. Then if $\vec{r_i}$ be displaced to $\vec{r_i}+d\vec{r_i}$, we have

$$d\vec{r_i} = \vec{u} + \vec{e} \times \vec{r_i},\ i = 1, 2, 3, \ldots$$

[Formula (F) § 2.3]

Now the algebraic sum of the virtual works is

$$\vec{P_1} \cdot d\vec{r_1} + \vec{P_2} \cdot d\vec{r_2} + \ldots$$

$$= \vec{P_1} \cdot (\vec{u} + \vec{e} \times \vec{r_1}) + \vec{P_2} \cdot (\vec{u} + \vec{e} \times \vec{r_2}) + \ldots$$

$$= (\vec{P_1} + \vec{P_2} + \ldots) \cdot \vec{u} + \vec{P_1} \cdot (\vec{e} \times \vec{r_1}) + \vec{P_2} \cdot (\vec{e} \times \vec{r_2}) + \ldots$$

$$= \vec{R} \cdot \vec{u} + \vec{e} \cdot (\vec{r_1} \times \vec{P_1}) + \vec{e} \cdot (\vec{r_2} \times \vec{P_2}) + \ldots$$

$$[\because\ \vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a})]$$

$$= \vec{u} \cdot \vec{R} + \vec{e} \cdot [\vec{r_1} \times \vec{P_1} + \vec{r_2} \times \vec{P_2} + \ldots]$$

$$= \vec{u} \cdot \vec{R} + \vec{e} \cdot \vec{G}, \quad \text{from (1) and (2)}$$

that is $$\Sigma \vec{P_i} \cdot d\vec{r_i} = \vec{u} \cdot \vec{R} + \vec{e} \cdot \vec{G}.$$

But, it is given that the given forces are in equilibrium

$$\Rightarrow \vec{R} = 0 \quad \text{and} \quad \vec{G} = 0$$

$$\Rightarrow \Sigma \vec{\mathbf{P}_i} \cdot d\vec{\mathbf{r}_i} = 0.$$ which proves.

statement and proof of the convers e of the principle of virtual work

Statement. If the algebraic sum of the virtual works done by any system of coplanar forces acting at different points of a rigid body be zero for any displacement whatever of the body consistent with the geometrical conditions of the system, then the system of forces is in equilibrium.

Proof. We have

$$\sum_i \vec{P_i} \cdot d\vec{r_i} = 0 \text{ for any displacement;}$$

that is, $\vec{u} \cdot \vec{R} + \vec{e} \cdot \vec{G} = 0$ for any displacement, .. (1)

where $\vec{R}$ is the single force and $\vec{G}$ is the moment of the couple, into which the given system of forces is compounded.

Let us first choose a displacement such that it is a translation $\vec{u}$ only.

So $$\vec{e}=0 \quad \text{and} \quad \vec{u} \neq 0.$$

Putting in (1), we have

$$\vec{u} \cdot \vec{R}=0 \Rightarrow \vec{R}=0, \qquad .. \quad (2)$$

Next we choose a displacement to be one of rotation $\vec{e}$ only (about O).

So $$\vec{e} \neq 0 \quad \text{and} \quad \vec{u}=0.$$

Hence (1) gives $$\vec{e} \cdot \vec{G}=0 \Rightarrow \vec{G}=0. \qquad .. \quad (3)$$

From (2) and (3), $\vec{R}=0$ and $\vec{G}=0$

$\Rightarrow$ the given system of forces is in equilibrium. Proved.